

Analytical-Equations to Generate the Free Electron Current Vector Field and the Angular-Momentum-Density Function

$$Y_0^0(\phi, \theta)$$

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Please consult the *Grand Unified Theory of Classical Physics* by Dr. Randell L. Mills. This file corresponds to Appendix IV.

Initialization Cells

Rotation of a Great Circle in the xy-Plane About the $(i_x, 0, i_z)$ -Axis by 2π

```

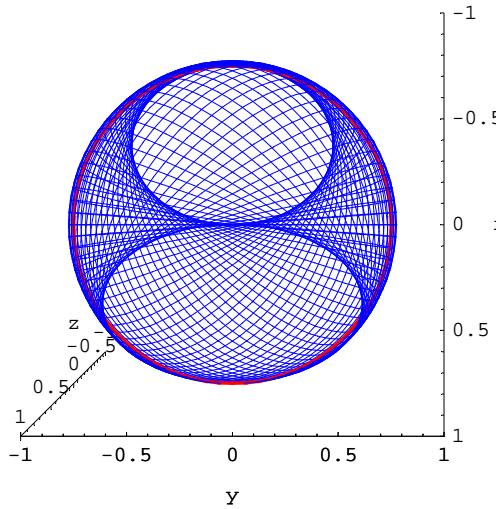
Clear[\theta]
Equ2 = yrot[-π/4].zrot[\theta].yrot[π/4];
MatrixForm[%]


$$\begin{pmatrix} \frac{1}{2} + \frac{\cos[\theta]}{2} & \frac{\sin[\theta]}{\sqrt{2}} & \frac{1}{2} - \frac{\cos[\theta]}{2} \\ -\frac{\sin[\theta]}{\sqrt{2}} & \cos[\theta] & \frac{\sin[\theta]}{\sqrt{2}} \\ \frac{1}{2} - \frac{\cos[\theta]}{2} & -\frac{\sin[\theta]}{\sqrt{2}} & \frac{1}{2} + \frac{\cos[\theta]}{2} \end{pmatrix}$$


ComponentFunct[Equ2, {R Cos[\phi], R Sin[\phi], 0}, {Red, Thickness[0.006]}, Blue];

```

```
Figure1 = Show[Array[p, 60], ViewPoint -> {0, 0, 2}, Axes -> True,
ViewVertical -> {-1, 0, 0}, AxesLabel -> {x, y, z}, DisplayFunction -> $DisplayFunction];
```



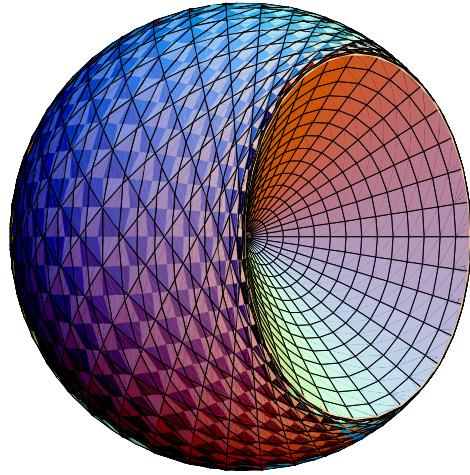
Conical Surfaces Formed by Variation of ρ

```
OS1 = SurfaceOfRevolution[{Cos[t], Sin[t], 0}, {t, \[Pi], 2 \[Pi]}, RevolutionAxis -> {1, 0, 1},
PlotRange -> All, SphericalRegion -> True, Boxed -> False, Axes -> False, ImageSize -> 72 * 6];

OS2 = SurfaceOfRevolution[{Cos[t], Sin[t], 0}, {t, 0, \[Pi]}, RevolutionAxis -> {1, 0, 1},
PlotRange -> All, SphericalRegion -> True, Boxed -> False, Axes -> False, ImageSize -> 72 * 6];

ConicalSect = SurfaceOfRevolution[{t, 0, 0}, {t, -1, 1}, RevolutionAxis -> {1, 0, 1},
PlotRange -> All, SphericalRegion -> True, Boxed -> False, Axes -> False, ImageSize -> 72 * 6];
```

```
Figure2 = Show[OS1, OS2, ConicalSect, ViewPoint -> {0, 0, 2}];
```



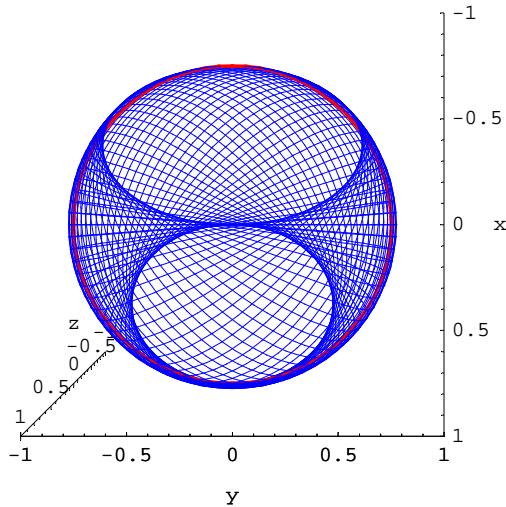
Rotation of a Great Circle in the xy-Plane About the $(-i_x, 0i_y, i_z)$ -Axis by 2π

```
Clear[\theta]
Equ7 = yrot[\frac{\pi}{4}] . zrot[\theta] . yrot[-\frac{\pi}{4}];
MatrixForm[%]


$$\begin{pmatrix} \frac{1}{2} + \frac{\cos[\theta]}{2} & \frac{\sin[\theta]}{\sqrt{2}} & -\frac{1}{2} + \frac{\cos[\theta]}{2} \\ -\frac{\sin[\theta]}{\sqrt{2}} & \cos[\theta] & -\frac{\sin[\theta]}{\sqrt{2}} \\ -\frac{1}{2} + \frac{\cos[\theta]}{2} & \frac{\sin[\theta]}{\sqrt{2}} & \frac{1}{2} + \frac{\cos[\theta]}{2} \end{pmatrix}$$


ComponentFunct[Equ7, {R Cos[\phi], R Sin[\phi], 0}, {Red, Thickness[0.006]}, Blue];
```

```
Figure3 = Show[Array[p, 60], ViewPoint -> {0, 0, 2}, Axes -> True, AxesLabel -> {x, y, z},
ViewVertical -> {-1, 0, 0}, DisplayFunction -> \$DisplayFunction];
```



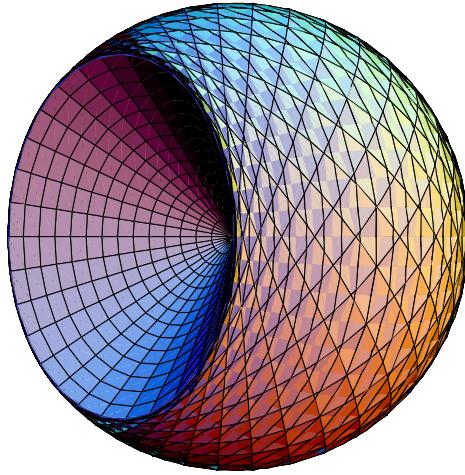
Conical Surfaces Formed by Variation of ρ

```
OS3 = SurfaceOfRevolution[{Cos[t], Sin[t], 0}, {t, \[Pi], 2 \[Pi]}, RevolutionAxis -> {-1, 0, 1},
PlotRange -> All, SphericalRegion -> True, Boxed -> False, Axes -> False, ImageSize -> 72 * 6];

OS4 = SurfaceOfRevolution[{Cos[t], Sin[t], 0}, {t, 0, \[Pi]}, RevolutionAxis -> {-1, 0, 1},
PlotRange -> All, SphericalRegion -> True, Boxed -> False, Axes -> False, ImageSize -> 72 * 6];

ConicalSect2 = SurfaceOfRevolution[{t, 0, 0}, {t, -1, 1}, RevolutionAxis -> {-1, 0, 1},
PlotRange -> All, SphericalRegion -> True, Boxed -> False, Axes -> False, ImageSize -> 72 * 6];
```

```
Figure4 = Show[OS3, OS4, ConicalSect2, ViewPoint -> {0, 0, 2}];
```



The Momentum-Density Function $Y_0^0(\phi, \theta)$

- Matrices to Visualize the Momentum-Density of $Y_0^0(\phi, \theta)$ for combined precession motion of the free electron about the $(i_x, 0, i_y, i_z)$ -Axis and Z-axis.

$$\text{Equl1} = \text{yrot}\left[-\frac{\pi}{4}\right] \cdot \{\{\rho \cos[\phi]\}, \{\rho \sin[\phi]\}, \{0\}\}$$

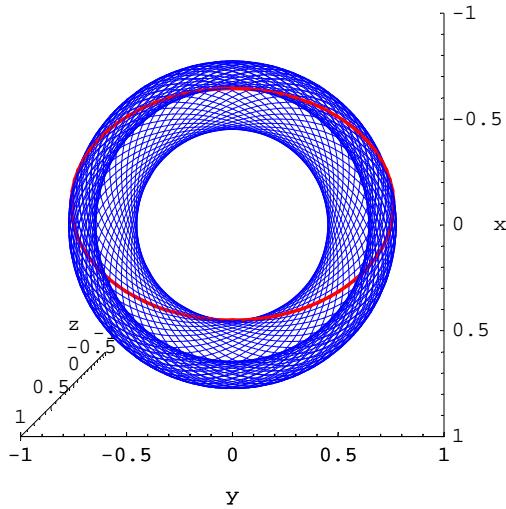
$$\left\{\left\{\frac{\rho \cos[\phi]}{\sqrt{2}}\right\}, \{\rho \sin[\phi]\}, \left\{-\frac{\rho \cos[\phi]}{\sqrt{2}}\right\}\right\}$$

```
Clear[\theta]
Equl2 = zrot[\theta];
MatrixForm[%]
```

$$\begin{pmatrix} \cos[\theta] & \sin[\theta] & 0 \\ -\sin[\theta] & \cos[\theta] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
ComponentFunct[Equ12, {R Cos[\phi], R Sin[\phi], -R Cos[\phi]}, {Red, Thickness[0.006]}, Blue];
```

```
Figure5 = Show[Array[p, 60], ViewPoint -> {0, 0, 2}, ViewVertical -> {-1, 0, 0},
Axes -> True, AxesLabel -> {x, y, z}, DisplayFunction -> $DisplayFunction];
```



■ Convolution Generation of $Y_0^0(\phi, \theta)$

```
Clear[m, M, n, NN]
θ := m (2 π / M);
MatrixForm[Equ12]
```

$$\begin{pmatrix} \cos\left(\frac{2m\pi}{M}\right) & \sin\left(\frac{2m\pi}{M}\right) & 0 \\ -\sin\left(\frac{2m\pi}{M}\right) & \cos\left(\frac{2m\pi}{M}\right) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Change theta to gamma in Equ 2:

$$\text{Equ2B} = \begin{pmatrix} \frac{1}{2} + \frac{\cos[\gamma]}{2} & \frac{\sin[\gamma]}{\sqrt{2}} & \frac{1}{2} - \frac{\cos[\gamma]}{2} \\ -\frac{\sin[\gamma]}{\sqrt{2}} & \cos[\gamma] & \frac{\sin[\gamma]}{\sqrt{2}} \\ \frac{1}{2} - \frac{\cos[\gamma]}{2} & -\frac{\sin[\gamma]}{\sqrt{2}} & \frac{1}{2} + \frac{\cos[\gamma]}{2} \end{pmatrix};$$

```

 $\gamma := n \frac{2\pi}{NN};$ 
MatrixForm[Equ2B]


$$\begin{pmatrix} \frac{1}{2} + \frac{1}{2} \cos[\frac{2n\pi}{NN}] & \frac{\sin[\frac{2n\pi}{NN}]}{\sqrt{2}} & \frac{1}{2} - \frac{1}{2} \cos[\frac{2n\pi}{NN}] \\ -\frac{\sin[\frac{2n\pi}{NN}]}{\sqrt{2}} & \cos[\frac{2n\pi}{NN}] & \frac{\sin[\frac{2n\pi}{NN}]}{\sqrt{2}} \\ \frac{1}{2} - \frac{1}{2} \cos[\frac{2n\pi}{NN}] & -\frac{\sin[\frac{2n\pi}{NN}]}{\sqrt{2}} & \frac{1}{2} + \frac{1}{2} \cos[\frac{2n\pi}{NN}] \end{pmatrix}$$


Clear[m, M, n, NN]
Equ18 = FullSimplify[Equ1.2.Equ2B]
MatrixForm[%]


$$\left\{ \left\{ \cos[\frac{2m\pi}{M}] \cos[\frac{n\pi}{NN}]^2 - \frac{\sin[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}]}{\sqrt{2}}, \cos[\frac{2n\pi}{NN}] \sin[\frac{2m\pi}{M}] + \frac{\cos[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}]}{\sqrt{2}}, \right. \right.$$


$$\left. \cos[\frac{2m\pi}{M}] \sin[\frac{n\pi}{NN}]^2 + \frac{\sin[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}]}{\sqrt{2}} \right\},$$


$$\left\{ -\cos[\frac{n\pi}{NN}]^2 \sin[\frac{2m\pi}{M}] - \frac{\cos[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}]}{\sqrt{2}}, \right.$$


$$\left. \cos[\frac{2m\pi}{M}] \cos[\frac{2n\pi}{NN}] - \frac{\sin[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}]}{\sqrt{2}}, \right.$$

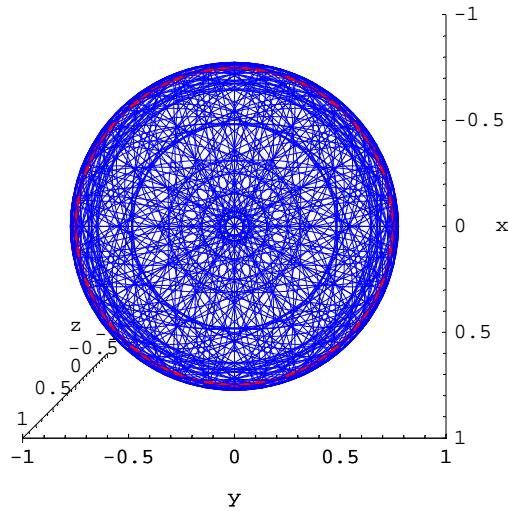

$$\left. -\sin[\frac{2m\pi}{M}] \sin[\frac{n\pi}{NN}]^2 + \frac{\cos[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}]}{\sqrt{2}} \right\}, \left\{ \sin[\frac{n\pi}{NN}]^2, -\frac{\sin[\frac{2n\pi}{NN}]}{\sqrt{2}}, \cos[\frac{n\pi}{NN}]^2 \right\}$$


$$\begin{pmatrix} \cos[\frac{2m\pi}{M}] \cos[\frac{n\pi}{NN}]^2 - \frac{\sin[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}]}{\sqrt{2}} & \cos[\frac{2n\pi}{NN}] \sin[\frac{2m\pi}{M}] + \frac{\cos[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}]}{\sqrt{2}} & \cos[\frac{2m\pi}{M}] \sin \\ -\cos[\frac{n\pi}{NN}]^2 \sin[\frac{2m\pi}{M}] - \frac{\cos[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}]}{\sqrt{2}} & \cos[\frac{2m\pi}{M}] \cos[\frac{2n\pi}{NN}] - \frac{\sin[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}]}{\sqrt{2}} & -\sin[\frac{2m\pi}{M}] \sin \\ \sin[\frac{n\pi}{NN}]^2 & -\frac{\sin[\frac{2n\pi}{NN}]}{\sqrt{2}} & \end{pmatrix}$$


ConvolutionFunct[Equ18, {R Cos[\phi], R Sin[\phi], 0}, {Red, Thickness[0.006]}, Blue];

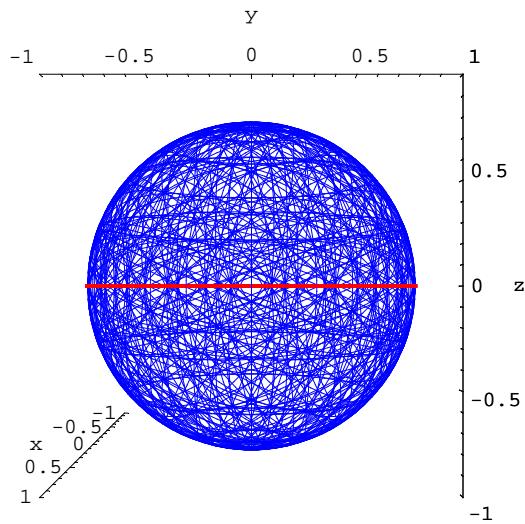
```

```
Figure6 = Show[Array[Component, {12}], ViewPoint -> {0, 0, 2},  
ViewVertical -> {-1, 0, 0}, Axes -> True, AxesLabel -> {x, y, z},  
PlotRange -> {{-1, 1}, {-1, 1}, {-1, 1}}, DisplayFunction -> $DisplayFunction];
```



Made this transverse perspective looking down the y axis.

```
Figure7 = Show[Figure6, ViewPoint -> {2, 0, 0}, ViewVertical -> {0, 0, 1}, Axes -> True];
```



■ Matrices to Visualize the Momentum-Density of $Y_0^0(\phi, \theta)$ for combined precession motion of the free electron about the $(-i_x, 0i_y, i_z)$ -Axis and Z-axis.

```

Equ19 = yrot[ $\frac{\pi}{4}$ ] . {{ρ Cos[φ]}, {ρ Sin[φ]}, {0}}
{{ $\frac{\rho \cos[\phi]}{\sqrt{2}}$ }, {ρ Sin[φ]}, { $\frac{\rho \cos[\phi]}{\sqrt{2}}$ }}

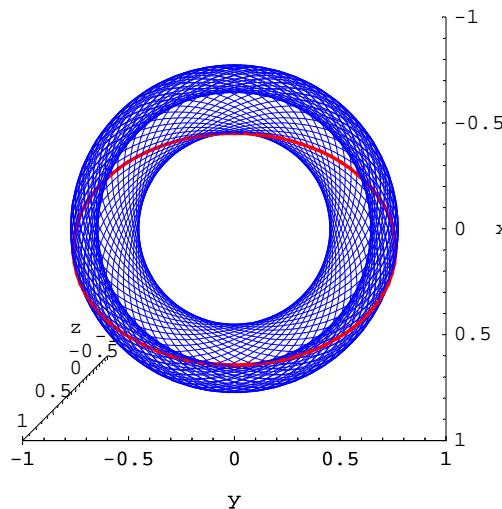
Clear[θ]
Equ20 = zrot[θ];
MatrixForm[%]


$$\begin{pmatrix} \cos[\theta] & \sin[\theta] & 0 \\ -\sin[\theta] & \cos[\theta] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$


ComponentFunct[Equ20, { $\frac{R \cos[\phi]}{\sqrt{2}}$ , R Sin[φ],  $\frac{R \cos[\phi]}{\sqrt{2}}$ }, {Red, Thickness[0.006]}, Blue];

Figure8 = Show[Array[p, 60], ViewPoint → {0, 0, 2}, ViewVertical → {-1, 0, 0},
Axes → True, AxesLabel → {x, y, z}, DisplayFunction → $DisplayFunction];

```



```

Clear[m, M, n, NN]
θ := m  $\frac{2\pi}{M}$ ;
MatrixForm[Equ20]


$$\begin{pmatrix} \cos[\frac{2m\pi}{M}] & \sin[\frac{2m\pi}{M}] & 0 \\ -\sin[\frac{2m\pi}{M}] & \cos[\frac{2m\pi}{M}] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$


```

Change theta to gamma in Equ 7:

$$\text{Equ7B} = \begin{pmatrix} \frac{1}{2} + \frac{\cos[\gamma]}{2} & \frac{\sin[\gamma]}{\sqrt{2}} & -\frac{1}{2} + \frac{\cos[\gamma]}{2} \\ -\frac{\sin[\gamma]}{\sqrt{2}} & \cos[\gamma] & -\frac{\sin[\gamma]}{\sqrt{2}} \\ -\frac{1}{2} + \frac{\cos[\gamma]}{2} & \frac{\sin[\gamma]}{\sqrt{2}} & \frac{1}{2} + \frac{\cos[\gamma]}{2} \end{pmatrix};$$

$$\gamma := n \frac{2\pi}{NN};$$

MatrixForm[Equ7B]

$$\begin{pmatrix} \frac{1}{2} + \frac{1}{2} \cos[\frac{2n\pi}{NN}] & \frac{\sin[\frac{2n\pi}{NN}]}{\sqrt{2}} & -\frac{1}{2} + \frac{1}{2} \cos[\frac{2n\pi}{NN}] \\ -\frac{\sin[\frac{2n\pi}{NN}]}{\sqrt{2}} & \cos[\frac{2n\pi}{NN}] & -\frac{\sin[\frac{2n\pi}{NN}]}{\sqrt{2}} \\ -\frac{1}{2} + \frac{1}{2} \cos[\frac{2n\pi}{NN}] & \frac{\sin[\frac{2n\pi}{NN}]}{\sqrt{2}} & \frac{1}{2} + \frac{1}{2} \cos[\frac{2n\pi}{NN}] \end{pmatrix}$$

Clear[m, M, n, NN]

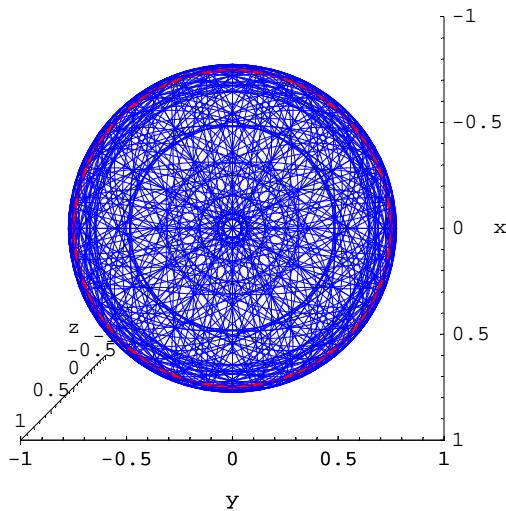
Equ23 = FullSimplify[Equ20.Equ7B]

MatrixForm[%]

$$\begin{aligned} & \left\{ \cos[\frac{2m\pi}{M}] \cos[\frac{n\pi}{NN}]^2 - \frac{\sin[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}]}{\sqrt{2}}, \cos[\frac{2n\pi}{NN}] \sin[\frac{2m\pi}{M}] + \frac{\cos[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}]}{\sqrt{2}}, \right. \\ & \quad \left. -\cos[\frac{2m\pi}{M}] \sin[\frac{n\pi}{NN}]^2 - \frac{\sin[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}]}{\sqrt{2}} \right\}, \\ & \left\{ -\cos[\frac{n\pi}{NN}]^2 \sin[\frac{2m\pi}{M}] - \frac{\cos[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}]}{\sqrt{2}}, \right. \\ & \quad \left. \cos[\frac{2m\pi}{M}] \cos[\frac{2n\pi}{NN}] - \frac{\sin[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}]}{\sqrt{2}}, \right. \\ & \quad \left. \sin[\frac{2m\pi}{M}] \sin[\frac{n\pi}{NN}]^2 - \frac{\cos[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}]}{\sqrt{2}} \right\}, \left\{ -\sin[\frac{n\pi}{NN}]^2, \frac{\sin[\frac{2n\pi}{NN}]}{\sqrt{2}}, \cos[\frac{n\pi}{NN}]^2 \right\} \} \\ & \left\{ \cos[\frac{2m\pi}{M}] \cos[\frac{n\pi}{NN}]^2 - \frac{\sin[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}]}{\sqrt{2}}, \cos[\frac{2n\pi}{NN}] \sin[\frac{2m\pi}{M}] + \frac{\cos[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}]}{\sqrt{2}}, -\cos[\frac{2m\pi}{M}] \sin[\frac{n\pi}{NN}]^2 \sin[\frac{2m\pi}{M}] - \frac{\cos[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}]}{\sqrt{2}}, \right. \\ & \quad \left. \cos[\frac{2m\pi}{M}] \cos[\frac{2n\pi}{NN}] - \frac{\sin[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}]}{\sqrt{2}}, \cos[\frac{2n\pi}{NN}] \cos[\frac{2m\pi}{M}] - \frac{\sin[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}]}{\sqrt{2}}, \sin[\frac{2m\pi}{M}] \sin[\frac{2n\pi}{NN}] \right. \\ & \quad \left. -\sin[\frac{n\pi}{NN}]^2 \right\} \end{aligned}$$

ConvolutionFunct[Equ23, {R Cos[\phi], R Sin[\phi], 0}, {Red, Thickness[0.006]}, Blue];

```
EquivalentFigure6 = Show[Array[Component, {12}], ViewPoint -> {0, 0, 2},  
ViewVertical -> {-1, 0, 0}, Axes -> True, AxesLabel -> {x, y, z},  
PlotRange -> {{-1, 1}, {-1, 1}, {-1, 1}}, DisplayFunction -> $DisplayFunction];
```



```
EquivalentFigure7 =  
Show[EquivalentFigure6, ViewPoint -> {2, 0, 0}, ViewVertical -> {0, 0, 1}, Axes -> True];
```

