Analytical-Equation Derivation of the Photon Electric and Magnetic Fields

This is a computational notebook to Appendix VI of R. Mills, *The Grand Unified Theory of Classical Quantum Mechanics*, January 2005 Edition, posted at: http://www.blacklightpower.com/bookdownload.shtml. This notebook has all needed equations. Equation numbers here correspond to the book, but some equations from the book have been omitted.

We will require the packages:

```
<< Graphics`ParametricPlot3D`
<< Graphics`Graphics3D`
<< Graphics`Colors`
<< Graphics`Shapes`
```

We will use the axis rotation matrices:

```
zrot[angle_] := ( Cos[angle]   Sin[angle]  0
                 -Sin[angle]   Cos[angle]  0
                      0            0       1 );
```

Component Function

The function ComponentFunct below will be used repeatedly to generate the figures. Input variables include the rotation matrix to be used (RotMatrix) the coordinates of the initial loop (FirstLoop), and the color of the loops in the series (LoopColor).

```
ComponentFunct[RotMatrix_, FirstLoop_, LoopColor_] :=
 (Steps := 15;
  R := 1;
  dθ := N[π/(Steps*2)];
  Clear[θ, loop, p, LoopCoords];
  Array[LoopCoords, {Steps, 2}];
  LoopCoords[1, 1] = FirstLoop;
  Array[loop, {Steps, 2, 3}];
  Array[p, {Steps}];
  Do[Do[LoopCoords[a, b] := {{0}, {0}, {0}}, {b, 2}], {a, 2, Steps}];
  Do[Do[Do[loop[d, f, g] := 0, {g, 3}], {f, 2}] loop[1, 1, 1], {d, 1, Steps}];

  θ := 0;
  {loop[1, 1, 1], loop[1, 1, 2], loop[1, 1, 3]} :=
   Evaluate[ReleaseHold[RotMatrix.LoopCoords[1, 1]]];

  p[1] = ParametricPlot3D[
    {loop[1, 1, 1], loop[1, 1, 2],
     loop[1, 1, 3], Evaluate[ReleaseHold[LoopColor]]}, {ϕ, 0, 2 π},
    Boxed → False,
    AspectRatio → Automatic, SphericalRegion → True,
    Axes → False, PlotRange → {{-1, 1}, {-1, 1}, {-1, 1}}];

  Do[
   θ := dθ * (i - 1);
   LoopCoords[i, 1] := Evaluate[ReleaseHold[RotMatrix.LoopCoords[1, 1]]];
   {loop[i, 1, 1], loop[i, 1, 2], loop[i, 1, 3]} :=
    Evaluate[ReleaseHold[LoopCoords[i, 1]]];

   p[i] = ParametricPlot3D[
     {loop[i, 1, 1], loop[i, 1, 2], loop[i, 1, 3], LoopColor}, {ϕ, 0, 2 π},
     Boxed → False,
     Axes → False,
     AspectRatio → Automatic, SphericalRegion → True],

   {i, 2, Steps}];)
```

Analytical Equations to Generate the Right-Handed Circularly Polarized Photon Electric and Magnetic Vector Field by Rotation of the Great-Circle Basis Elements about the $(i_x, i_y, 0i_z)$-Axis by $\frac{\pi}{2}$.

The right-handed circularly polarized (RHCP) photon electric and magnetic vector field (photon-e&mvf) is also generated following a similar procedure as that used to generate the orbitsphere in Appendix III: Analytical Equations to Generate the Orbitsphere Current-Vector Field and the Uniform Current (Charge)-Density Function $Y_0{}^0(\phi, \theta)$. The RHCP photon-e&mvf is generated by the rotation of the basis elements comprising the great circle magnetic field line in the xz-plane and the great circle electric field line in the yz-plane about the $(i_x, i_y, 0i_z)$-axis by $\frac{\pi}{2}$. A first transformation matrix is generated by the combined rotation of the great circles about the z-axis by $\frac{\pi}{4}$ then about the x-axis by θ where positive rotations about an axis are defined as clockwise:

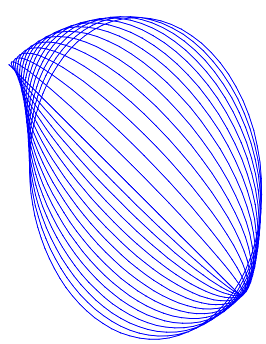
```
Clear[θ];
```

$$\texttt{Equ1} := \begin{pmatrix} \text{Cos}[\frac{\pi}{4}] & \text{Sin}[\frac{\pi}{4}] & 0 \\ -\text{Sin}[\frac{\pi}{4}]\,\text{Cos}[\theta] & \text{Cos}[\frac{\pi}{4}]\,\text{Cos}[\theta] & \text{Sin}[\theta] \\ \text{Sin}[\frac{\pi}{4}]\,\text{Sin}[\theta] & -\text{Cos}[\frac{\pi}{4}]\,\text{Sin}[\theta] & \text{Cos}[\theta] \end{pmatrix};$$

```
ComponentFunct[Equ1, {R Cos[ϕ], 0, R Sin[ϕ]}, Blue];
```

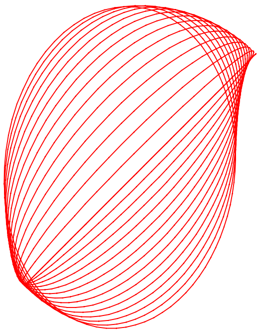
The transformation matrix about $(i_x, i_y, 0i_z)$ is given by multiplication of the output of the matrix given by Eq. (1) by the matrix corresponding to a rotation about the z-axis of $-\frac{\pi}{4}$. The output of the matrix given by Eq. (1) is shown in Figure 1 (consisting of parts A-C in the *Mathematica* notebook) wherein θ is varied from 0 to $\frac{\pi}{2}$. Shown with 6 degree increments of the angle θ. (Electric field lines red; Magnetic field lines blue).

```
Fig1A = Show[Array[p, {15}], ViewPoint → {0, 0, 2}];
```

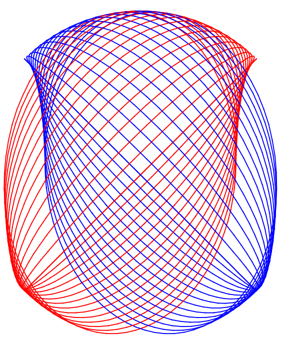


```
Clear[θ];
ComponentFunct[Equ1, {0, R Cos[ϕ], R Sin[ϕ]}, Red];
```

```
Fig1B = Show[Array[p, {15}], ViewPoint → {0, 0, 2}];
```



```
Fig1C = Show[Fig1A, Fig1B, ViewPoint → {0, 0, 2}];
```



The rotation matrix about the z-axis by $-\frac{\pi}{4}$, is given by "zrotneg." Thus,

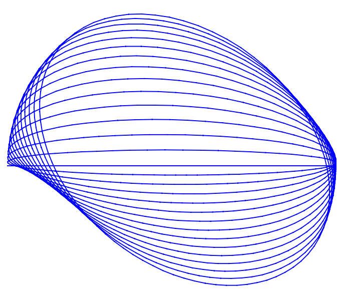
```
Clear[θ]
Equ5 = zrot[-π/4].Equ1;
MatrixForm[%]
```

$$\begin{pmatrix} \frac{1}{2}+\frac{\text{Cos}[\theta]}{2} & \frac{1}{2}-\frac{\text{Cos}[\theta]}{2} & -\frac{\text{Sin}[\theta]}{\sqrt{2}} \\ \frac{1}{2}-\frac{\text{Cos}[\theta]}{2} & \frac{1}{2}+\frac{\text{Cos}[\theta]}{2} & \frac{\text{Sin}[\theta]}{\sqrt{2}} \\ \frac{\text{Sin}[\theta]}{\sqrt{2}} & -\frac{\text{Sin}[\theta]}{\sqrt{2}} & \text{Cos}[\theta] \end{pmatrix}$$

```
ComponentFunct[Equ5, {R Cos[ϕ], 0, R Sin[ϕ]}, Blue];
```

The RHCP photon-e&mvf that is generated by the rotation of the great-circle basis elements in the xz- and yz-planes about the $(i_x,i_y,0i_z)$-axis by $\frac{\pi}{2}$ corresponding to the output of the matrix given by Eq. (5) is shown in Figure 2.

```
EFieldRHCP = Show[Array[p, {Steps}], ViewPoint → {0, 0, 2}];
```



```
Clear[θ];
ComponentFunct[Equ5, {0, R Cos[ϕ], R Sin[ϕ]}, Red];
```

```
BFieldRHCP = Show[Array[p, {Steps}], ViewPoint → {0, 0, 2}];
```

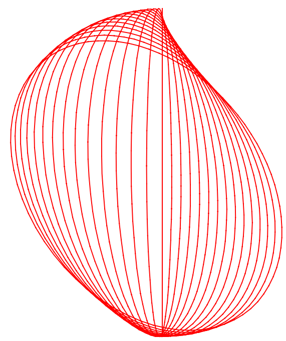
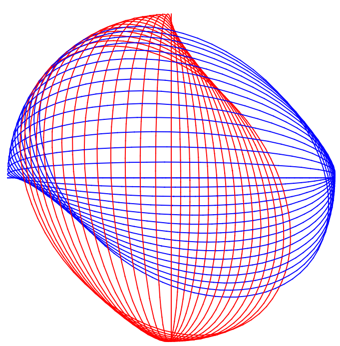
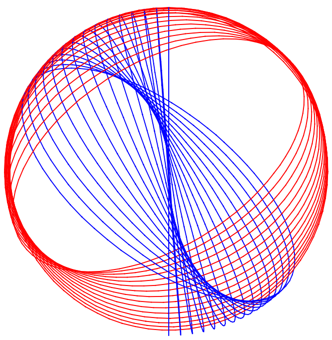


Figure 2 (A-C). The field-line pattern given by Eq. (5) from three orthogonal perspectives (z, x, y respectively) of a RHCP photon-e&mvf corresponding to the first great circle magnetic field line and the second great circle electric field line shown with 6 degree increments of the angle . (Electric field lines red; Magnetic field lines blue).

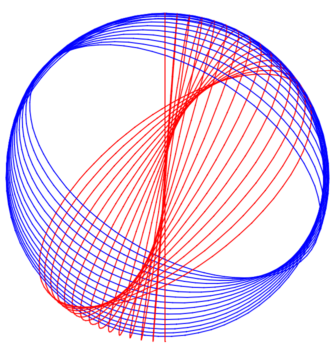
```
Fig2A = Show[BFieldRHCP, EFieldRHCP];
```



```
Fig2B = Show[RotateShape[Fig2A, π / 2, 0, 0], ViewPoint → {0, 2, 0}];
```



```
Fig2C = Show[Fig2A, ViewPoint → {0, 2, 0}];
```



Analytical Equations to Generate the Left-Handed Circularly Polarized Photon Electric and Magnetic Vector Field by Rotation of the Great-Circle Basis Elements about the (i_x,$-i_y$,$0i_z$)-Axis by $\frac{\pi}{2}$.

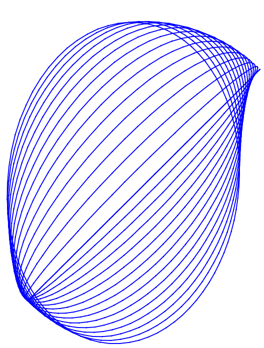
The left-handed circularly polarized (LHCP) photon electric and magnetic vector field (photon-e&mvf) is also generated following a similar procedure as that used to generate the orbitsphere in Appendix III: Analytical Equations to Generate the Orbitsphere Current Vector Field and the Uniform Current (Charge)-Density Function $Y_0{}^0(\phi, \theta)$. The LHCP photon-e&mvf is generated by the rotation of the basis elements comprising the great circle magnetic field line in the xz-plane and the great circle electric field line in the yz-plane about the (i_x,$-i_y$,$0i_z$)-axis by $\frac{\pi}{2}$. A first transformation matrix is generated by the combined rotation of the great circles about the z-axis by $\frac{\pi}{4}$ then about the x-axis by θ where positive rotations about an axis are defined as clockwise:

```
Clear[θ]
Equ6 := ( Cos[π/4]              -Sin[π/4]              0
          Sin[π/4] Cos[θ]       Cos[π/4] Cos[θ]        Sin[θ]
          -Sin[π/4] Sin[θ]      -Cos[π/4] Sin[θ]       Cos[θ] );
ComponentFunct[Equ6, {R Cos[ϕ], 0, R Sin[ϕ]}, Blue];
```

The transformation matrix about (i_x,$-i_y$,$0i_z$) is given by multiplication of the output of the matrix given by Eq. (6) by the matrix corresponding to a rotation about the z-axis of $\frac{\pi}{4}$. The output of the matrix given by Eq. (6) is shown in Figure 3 (consisting of parts A-C in the *Mathematica* notebook) wherein θ is varied from 0 to $\frac{\pi}{2}$.

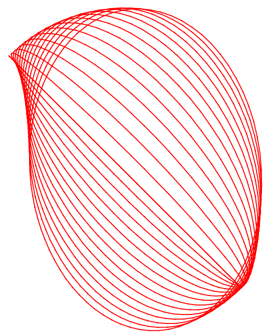
Figure 3. The electric field-line pattern given by Eq. (6) shown with 6 degree increments of the angle θ. (Electric field lines red; Magnetic field lines blue).

```
Fig3A = Show[Array[p, {Steps}], ViewPoint → {0, 0, 2}];
```

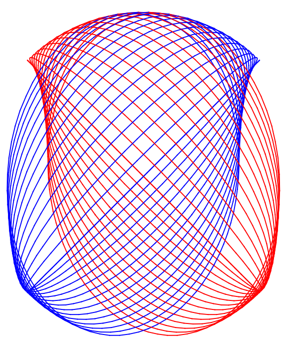


```
Clear[θ];
ComponentFunct[Equ6, {0, R Cos[ϕ], R Sin[ϕ]}, Red];
```

```
Fig3B = Show[Array[p, {Steps}], ViewPoint → {0, 0, 2}];
```



```
Fig3C = Show[Fig3A, Fig3B];
```



The rotation matrix about the z-axis by $-\frac{\pi}{4}$, is given by "zrotneg." Thus,

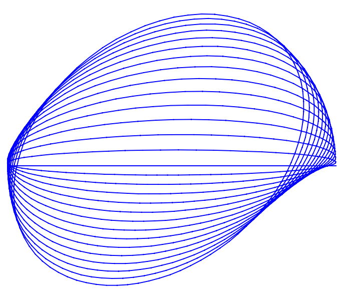
```
Clear[θ]
Equ10 = zrot[π/4].Equ6;
MatrixForm[%]
```

$$\begin{pmatrix} \frac{1}{2}+\frac{\text{Cos}[\theta]}{2} & -\frac{1}{2}+\frac{\text{Cos}[\theta]}{2} & \frac{\text{Sin}[\theta]}{\sqrt{2}} \\ -\frac{1}{2}+\frac{\text{Cos}[\theta]}{2} & \frac{1}{2}+\frac{\text{Cos}[\theta]}{2} & \frac{\text{Sin}[\theta]}{\sqrt{2}} \\ -\frac{\text{Sin}[\theta]}{\sqrt{2}} & -\frac{\text{Sin}[\theta]}{\sqrt{2}} & \text{Cos}[\theta] \end{pmatrix}$$

```
ComponentFunct[Equ10, {R Cos[ϕ], 0, R Sin[ϕ]}, Blue];
```

The LHCP photon-e&mvf that is generated by the rotation of the great-circle basis elements in the xz- and yz-planes about the $(i_x,-i_y,0i_z)$-axis by $\frac{\pi}{2}$ corresponding to the output of the matrix given by Eq. (10) is shown in Figure 4.

```
EFieldLHCP = Show[Array[p, {Steps}], ViewPoint → {0, 0, 2}];
```



```
Clear[θ]
ComponentFunct[Equ10, {0, R Cos[ϕ], R Sin[ϕ]}, Red];
```

```
BFieldLHCP = Show[Array[p, {Steps}], ViewPoint → {0, 0, 2}];
```

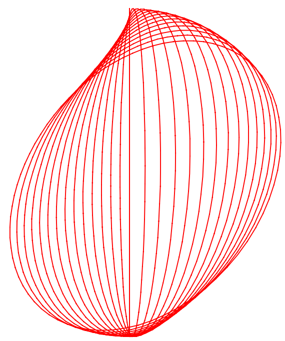
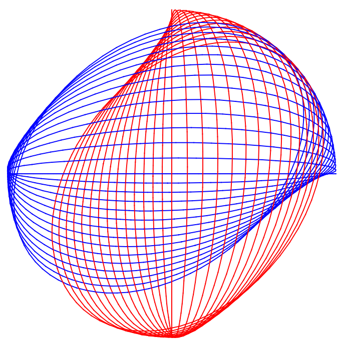
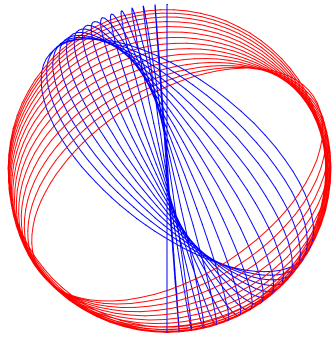


Figure 4. The field-line pattern given by Eq. (10) from three orthogonal perspectives (consisting of parts A-C in the *Mathematica* notebook) of a LHCP photon-e&mvf corresponding to the first great circle magnetic field line and the second great circle electric field line shown with 6 degree increments of the angle θ. (Electric field lines red; Magnetic field lines blue).

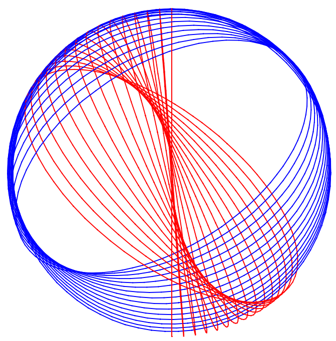
```
Fig4A = Show[BFieldLHCP, EFieldLHCP];
```



```
Fig4B = Show[RotateShape[Fig4A, π / 2, 0, 0], ViewPoint → {0, 2, 0}];
```



```
Fig4C = Show[Fig4A, ViewPoint → {0, 2, 0}];
```



Generation of the Linearly Polarized Photon Electric and Magnetic Vector Field

Figure 5. The field-line pattern given by Eqs. (5) and (10) from three orthogonal perspectives of a LP photon-e&mvf corresponding to the first great circle magnetic field line and the second great circle electric field line shown with 6 degree increments of the angle θ about each of the $(i_x, i_y, 0i_z)$- and $(i_x, -i_y, 0i_z)$-axes. (Electric field lines red; Magnetic field lines blue).

```
Fig5 = Show[Fig2A, Fig4A];
```